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Reentrant Phase Transitions in Ferronematic Liquid Crystals

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Within the continuum theory we study the influence of codirectional electric and magnetic fields on the orientational structure of a ferronematic layer taking into account the segregation effects. Recently in [Phys. Rev. E 81, 051710 (2010)] the tricritical behavior of the magnetic Freedericksz transition in ferronematics has been theoretically predicted. This paper shows that an external electric field can change the character of this transition. We find reentrant Freedericksz transitions in ferronematic liquid crystals subjected to the electric and magnetic fields. These transitions can be either first- or second-order ones. We derive the analytical expression for the width of the area of the reentrant orientational transitions.

Keywords ferronematic; reentrant phase transitions; Freedericksz transition; magnetic and electric fields

1. Introduction

Ferronematic liquid crystals, or ferronematics (FNs), are dilute suspensions of anisometric single-domain magnetic particles based on nematic liquid crystals (NLCs) [1, 2]. Due to liquid-crystalline matrix, FNs possess high fluidity and anisotropy of physical properties, while the presence of ferroparticles in the matrix leads to strong magnetic response of the suspension. These features allow ferronematics to belong to the unique class of soft magnetic materials with controlled physical properties. Studies of such media are the basis of understanding the molecular and supramolecular structure of both existing and future smart materials. Ferronematic liquid crystals are artificial soft condensed matters, thereby actual research problems include synthesis, experimental and theoretical study of their physical properties and behavior in external fields. In the last few years an increased interest in these materials have been seen from both experimenters [3–8] and theoreticians [9–14].

Recently in [14] the tricritical behavior of the magnetic Freedericksz transition in ferronematic liquid crystals has been theoretically predicted. In the present paper we show that an external electric field can change the character of magnetic transition. We study the combined influence of electric and magnetic fields on the orientational structure of a ferronematic layer taking into account the segregation effects. The anisotropies of a magnetic susceptibility and dielectric permeability are considered to be positive. We impose

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rigid planar coupling conditions of the director at the layer boundaries and soft orthogonal coupling between the director and the magnetization.

The paper is organized as follows. In Sec. 2 we define the problem and derive the equations describing orientational structure of ferronematic layer under the combined influence of magnetic and electric fields taking into account the magnetic segregation effect. Threshold fields and reentrant phase transitions are considered in Sec. 3. In Sec. 4 we discuss the influence of the electric field on the character of the magnetic-field-induced Freedericksz transition in a ferronematic layer and present the orientational and concentrational distributions within the layer. The conclusions are listed in Sec. 5.

2. Equilibrium equations

Let us consider the ferronematic liquid crystal layer of thickness D sandwiched between two parallel plates (see Fig. 1). We introduce rectangular coordinate system with the x axis parallel and the z axis perpendicular to the plates. The coordinate origin is chosen in the middle of the layer. We impose rigid planar boundary conditions on the director $\mathbf{n}$, i.e., the director is fixed on the plates and the easy axis $\mathbf{n_0}$ is parallel to the x axis. Let the electric $\mathbf{E} = (0,\ 0,\ E)$ and magnetic $\mathbf{H} = (0,\ 0,\ H)$ fields are applied perpendicularly to the layer. The bulk free-energy density F_V of a ferronematic (SI units) in a magnetic and an electric fields has the form [1, 15]

$$F_V = F_1 + F_2 + F_3 + F_4 + F_5 + F_6, \tag{1}$$

$$F_1 = \frac{1}{2}[K_1(\nabla \cdot \mathbf{n})^2 + K_2(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + K_3(\mathbf{n} \times \nabla \times \mathbf{n})^2],$$

$$F_2 = -\frac{1}{2}\mu_0\chi_a(\mathbf{n} \cdot \mathbf{H})^2, \qquad F_3 = -\mu_0 M_s f\, \mathbf{m} \cdot \mathbf{H},$$

$$F_4 = \frac{k_B T}{v} f \ln f, \qquad F_5 = \frac{w}{d} f(\mathbf{n} \cdot \mathbf{m})^2,$$

$$F_6 = -\frac{1}{2}\varepsilon_0\varepsilon_a(\mathbf{n} \cdot \mathbf{E})^2.$$

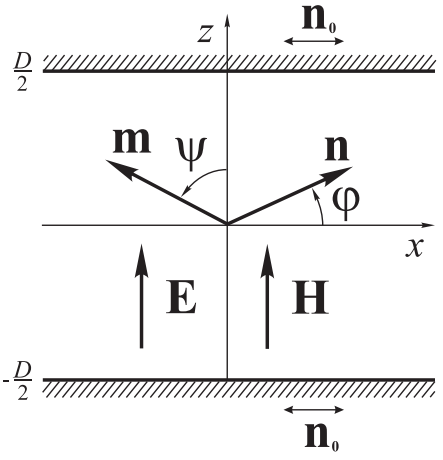

Figure 1. Ferronematic layer in electric **E** and magnetic **H** fields.

Here, K_1, K_2, and K_3 are elastic modules of NLC (Frank constants); χ_a and ε_a are the anisotropies of a magnetic susceptibility and dielectric permeability of NLC; M_s is the saturation magnetization of a ferroparticle material; μ_0 is the permeability of free space; ε_0 is the permittivity of free space; $f(\mathbf{r})$ is the local volume fraction of the particles in a suspension; $\mathbf{m}$ is the unit vector along the magnetization of a suspension; ν is the volume of a magnetic particle; k_B is the Boltzmann constant; d is the transverse diameter of a particle; T is the temperature; and w is the surface energy density of a coupling between NLC molecules and magnetic particles. The value w is chosen positive, so that in the absence of external force fields the free energy is minimal at $\mathbf{m}\perp\mathbf{n}$ that corresponds to the orthogonal coupling between the director and the magnetization. Further we assume that $\chi_a > 0$ and $\varepsilon_a > 0$.

The first term F_1 in Eq. (1) represents the bulk free-energy density of the director field elastic deformations of NLC (the Oseen-Frank potential). The second F_2 and the third F_3 contributions characterize the interactions of diamagnetic NLC matrix (the quadrupole magnetic mechanism of interaction) and magnetic moments of the particles (the dipole mechanism of interaction) with an external magnetic field $\mathbf{H}$, respectively. The fourth term F_4 is the contribution of the mixing entropy of the ideal ferromagnetic particle solution. The fifth term F_5 determines the surface coupling energy of magnetic particles with NLC matrix, and the last term F_6 characterizes the interaction of dielectric NLC matrix (the quadrupole electric mechanism of interaction) with an external electric field $\mathbf{E}$. We consider FN with low volume fraction $f \ll 1$ of ferroparticles and therefore we neglect the interparticle magnetic dipole-dipole interaction in a suspension.

The director $\mathbf{n}$ and the unit magnetization vector $\mathbf{m}$ can be written in the following form

$$\begin{aligned} \mathbf{n} &= [\cos\varphi(z),\ 0,\ \sin\varphi(z)], \\ \mathbf{m} &= [-\sin\psi(z),\ 0,\ \cos\psi(z)]. \end{aligned} \tag{2}$$

We choose the thickness D as the unit of length and introduce the dimensionless coordinate $\widetilde{z} = z/D$. Hereafter, for simplicity, we omit the squiggle over the dimensionless variables in equations. Let us define the following dimensionless parameters:

$$\begin{aligned} &h = HD\sqrt{\frac{\mu_0\chi_a}{K_1}}, \qquad b = M_s\bar{f}D\sqrt{\frac{\mu_0}{\chi_a K_1}}, \qquad k = \frac{K_3}{K_1}, \qquad \varkappa = \frac{k_B T\bar{f}D^2}{K_1\nu}, \\ &e = ED\sqrt{\frac{\varepsilon_0\varepsilon_a}{K_1}}, \qquad \sigma = \frac{w\bar{f}D^2}{K_1 d}, \qquad \bar{f} = \frac{N\nu}{V}. \end{aligned} \tag{3}$$

Here, $\bar{f}$ is the average volume fraction of magnetic particles in FN. When magnetic particles are uniformly distributed within the layer, the local volume fraction $f(z) \equiv \bar{f}$. Parameter h is the dimensionless magnetic field strength. As the unit of strength value $H_q = D^{-1}\sqrt{K_1/(\mu_0\chi_a)}$ is chosen. It is defined from the balance of the elastic deformations energy F_1 and diamagnetic contribution F_2 [see Eq. (1)]. At $H \gtrsim H_q$ the orientational distortions arise due to diamagnetic anisotropy of NLC matrix (the quadrupole mechanism of interaction). The similar comparison of the elastic F_1 and the dipole F_3 contributions results in the other typical quantity of the magnetic field strength $H_d = K_1/(\mu_0 M_s\bar{f}D^2)$. In this case for $H \gtrsim H_d$ the distortions in ferronematic are induced by the interaction of magnetic particles with an external magnetic field (the dipole mechanism). The parameter $b = H_q/H_d$ represents the ratio of fields H_q and H_d mentioned above and therefore

characterizes the mode of magnetic field influence on the FN [16]. For $b \gg 1$ ($H_d \ll H_q$) the distortions of the orientational structure in low fields are realized due to dipole mechanism; otherwise, for $b \ll 1$ ($H_q \ll H_d$) they are realized due to quadrupole mechanism. The mechanism of magnetic field influence on the FN is changed from the dipole to quadrupole one (and vice versa) when the contributions F_2 and F_3 to the free energy are of the same order of magnitude, that is, at $H \approx H_0 = M_s \bar{f}/\chi_a$.

Parameter e is the dimensionless electric field strength. As the unit of strength the electric Freedericksz field $E_q = D^{-1}\sqrt{K_1/(\varepsilon_0 \varepsilon_a)}$ in pure nematic liquid crystal is chosen [18].

The so-called segregation parameter [16] $\varkappa = (D/\lambda)^2$ is the square of the ratio of two typical lengths, namely, the layer thickness D and the segregation length $\lambda = (\nu K_1/k_B T \bar{f}\,)^{1/2}$. The latter gives a scale of stratification region in a ferronematic sample. Thus, for $\varkappa \gg 1$ the segregation effects are weak and the distribution of magnetic particles within the layer is close to uniform. In the opposite case $\varkappa < 1$ the inhomogeneity of magnetic particles redistribution in the layer is significant.

We also introduce the dimensionless coupling energy σ between magnetic particles and NLC matrix, and the ratio of Frank elastic constants k.

The free energy $F = \int_V F_V \, dV$ of a ferronematic layer is determined by expression (1) and in dimensionless form is given by

$$\widetilde{F} = \frac{D}{K_1 S} F = \int_{-1/2}^{1/2} \left[\frac{1}{2} \mathcal{K}(\varphi) \left(\frac{d\varphi}{dz} \right)^2 - bhg \cos\psi - \right.$$
$$\left. - \frac{1}{2}(h^2 + e^2) \sin^2\varphi + \varkappa g \ln g + \sigma g \sin^2(\varphi - \psi) \right] dz, \tag{4}$$

where S is the surface area of the bounding plates; besides we use the following notations:

$$g(z) = f(z)/\bar{f}, \tag{5}$$

$$\mathcal{K}(\varphi) = \cos^2\varphi + k \sin^2\varphi. \tag{6}$$

Here, $g(z)$ is the reduced volume fraction. Minimization of free energy (4) with respect to $\varphi(z)$ and $\psi(z)$ gives the equations for the director and the magnetization angles:

$$\mathcal{K}(\varphi)\,\varphi'' + \frac{1}{2}\frac{d\mathcal{K}(\varphi)}{d\varphi}\,\varphi'^2 + \frac{1}{2}(h^2 + e^2)\sin 2\varphi - \sigma g \sin 2(\varphi - \psi) = 0, \tag{7}$$

$$bh \sin\psi - \sigma \sin 2(\varphi - \psi) = 0, \tag{8}$$

subject to the boundary conditions

$$\varphi(-1/2) = \varphi(1/2) = 0, \tag{9}$$

corresponding to the rigid planar coupling of the director $\mathbf{n}$ on the surfaces. Hereinafter the prime denotes the derivative with respect to the dimensionless coordinate z.

The equilibrium distribution $g(z)$ of magnetic particles in the ferronematic layer can be found from the minimum of the free energy (4) with respect to $f(z)$ at a constant number of particles in suspension $\int f dV = N\nu$, that is

$$g = Q\,\mathcal{E}(\varphi, \psi), \qquad Q^{-1} = \int_{-1/2}^{1/2} \mathcal{E}(\varphi, \psi)\, dz, \tag{10}$$

where

$$\mathcal{E}(\varphi, \psi) = \exp\left\{\frac{bh}{\varkappa}\cos\psi - \frac{\sigma}{\varkappa}\sin^2(\varphi - \psi)\right\}. \tag{11}$$

The so-called bonding equation (8) determines the mutual orientation of the director and the magnetization [17], and Eq. (10) describes the magnetic segregation effect [1] in FNs, that is, a magnetic field **H** stimulates the redistribution of the magnetic particles in the layer, and so the concentration of the particles increases in those parts of the sample, where the sum of their magnetic energy in the field **H** and orientational energy in NLC matrix is minimum. For $\varkappa \to \infty$ the magnetic segregation effects become negligible. In this case, as is seen from Eqs. (10) and (11), the normalization coefficient $Q \to 1$ and the local concentration $f(z) \to \bar{f}$, i.e, $g(z) \to 1$.

Let us estimate the dimensionless quantities (3), using typical material parameters of NLCs [18] and magnetic particles [19]. Assuming for a ferronematic based on a liquid crystal 5CB (SI units) $\chi_a = 2.1 \times 10^{-6}$, $K_1 = 6.4 \times 10^{-12}$ N, $K_3 = 1.0 \times 10^{-11}$ N, $T = 298$ K, $\bar{f} = 2.0 \times 10^{-7}$, $M_s = 5 \times 10^5$ A·m^{-1}, $w = 10^{-6} - 10^{-4}$ N·m^{-1}, $d = 7.5 \times 10^{-8}$ m, $\nu = 8.8 \times 10^{-22}$ m^3, and supposing the layer thickness $D = 250\,\mu$m, we obtain $k \approx 1$, $\sigma \approx 10^{-2} - 1$, $b \approx 10$, and $\varkappa \approx 10^{-2}$. As is seen from these estimations, the smallness of $\varkappa$ indicates the importance of the magnetic segregation effects in the problem.

When the orientations of the electric **E** and magnetic **H** fields are orthogonal to the ferronematic layer (see Fig. 1), Eqs. (7)–(10) admit the uniform solution $\varphi(z) = \psi(z) = 0$, corresponding to the planar texture of the FN with orthogonal coupling between the director and the magnetization ($\mathbf{n} \perp \mathbf{m} || \mathbf{H}$).

Let us consider the nonuniform solutions for the director and magnetization within the layer. At first we multiply Eq.(7) by φ', Eq. (8) by $g\psi'$, and subtract the latter from the former:

$$\frac{d}{dz}[\mathcal{K}(\varphi)\,\varphi'^2 - (h^2 + e^2)\cos^2\varphi + 2\varkappa g] = 0. \tag{12}$$

In the middle of the layer the deviation of the director is maximum, that is, $\varphi' = 0$ at $z = 0$. Thus, the first integral of Eq. (12) has the form

$$\mathcal{K}^{1/2}(\varphi)\,\varphi' = \pm[(h^2 + e^2)(\cos^2\varphi - \cos^2\varphi_0) + 2\varkappa(g_0 - g)]^{1/2}. \tag{13}$$

Here, $g_0 \equiv g(\varphi_0, \psi_0)$, $\varphi_0 \equiv \varphi(0)$, and $\psi_0 \equiv \psi(0)$ are the distribution function of the particles, the angles of the director and the magnetization rotation in the middle of the layer, respectively.

Integration of Eq. (13) for $z > 0$ subject to the boundary conditions (9) gives

$$\frac{1}{2} - z = \pm\int_0^{\varphi(z)} \mathcal{R}^{1/2}(\varphi, \psi)\,d\varphi, \tag{14}$$

where

$$\mathcal{R}(\varphi, \psi) = \frac{\mathcal{K}(\varphi)}{(h^2 + e^2)(\cos^2\varphi - \cos^2\varphi_0) + 2\varkappa(g_0 - g)}, \tag{15}$$

and $\mathcal{K}(\varphi)$ is defined by Eq. (6). In the upper half-space of the layer ($z \geq 0$) the sign "+" gives solutions describing the counterclockwise director rotation ($\varphi > 0$), and the sign "−" corresponds to the clockwise rotation of the director ($\varphi < 0$).

In the middle of the layer ($z = 0$) the angle $\varphi = \varphi_0$; therefore, Eq. (14) reduces to

$$\frac{1}{2} = \pm \int_0^{\varphi_0} \mathcal{R}^{1/2}(\varphi, \psi)\, d\varphi. \tag{16}$$

Transforming the coefficient Q in Eq. (10) by means of expression (13), we obtain the equation for the distribution function $g(z)$,

$$\frac{1}{2} = \pm \int_0^{\varphi_0} g\mathcal{R}^{1/2}(\varphi, \psi)\, d\varphi. \tag{17}$$

Thus, Eqs. (14)–(17) in conjunction with bonding equation (8) determine the angles $\varphi(z)$ and $\psi(z)$ of the director and the magnetization rotation, the distribution function $g(z) = f(z)/\bar{f}$ in the FN layer as functions of the magnetic h and electric e field strengths, the coupling energy σ, the elastic anisotropy coefficient k, the dimensionless parameter b, and the segregation parameter $\varkappa$.

3. Threshold fields

When the electric $\mathbf{E} = (0,\ 0,\ E)$ and magnetic $\mathbf{H} = (0,\ 0,\ H)$ fields are orthogonal to the ferronematic layer, equations (14)–(17) admit the uniform solution $\varphi(z) = \psi(z) = 0$ corresponding to the planar texture of the FN with orthogonal coupling between the director and the magnetization ($\mathbf{n}\perp\mathbf{m}||\mathbf{H}||\mathbf{E}$). This state becomes unstable when the field h exceeds some threshold value [14]. Let us determine this threshold magnetic field h_c assuming that the layer is subjected to the electric field e. Close to h_c the angles φ and ψ are small, so the solutions can be expanded as a power series in the small parameter $\epsilon \ll 1$,

$$\varphi = \varphi_1\epsilon + \varphi_2\epsilon^2 + \cdots, \quad \psi = \psi_1\epsilon + \psi_2\epsilon^2 + \cdots,$$
$$h = h_c + h_1\epsilon + h_2\epsilon^2 + \cdots.$$

Substituting these expansions in Eqs. (7)–(10) in the lowest order we obtain

$$\varphi_1'' + \left(h_c^2 + e^2 - sbh_c\right)\varphi_1 = 0, \quad \psi_1 = s\,\varphi_1, \tag{18}$$

where

$$s \equiv 2\sigma/(2\sigma + bh_c), \quad 0 \le s \le 1. \tag{19}$$

Equations (18) with boundary conditions (9) have nontrivial solutions at $h_c^2 + e^2 > sbh_c$ only. Conditions for the existence of nontrivial solutions gives the equation for the threshold field $h_c = h_c(e, \sigma, b)$

$$h_c^2 + e^2 = \pi^2 + sbh_c. \tag{20}$$

This expression defines the transition in a ferronematic layer under the combined action of electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields. Let us rewrite Eq. (20) in terms of threshold electric field $e_c = e_c(h, \sigma, b)$

$$e_c = \sqrt{\pi^2 - h^2 + \frac{2\sigma bh}{2\sigma + bh}}. \tag{21}$$

In the absence of magnetic field ($h = 0$) threshold electric field e_c in FN coincides with a critical Freedericksz field $e_c^{LC} = \pi$ in pure LC [2]. In this case, as it should be, the magnetic particles are a passive admixture, which does not affect the transition. The phase diagrams of the orientational transitions in a FN layer, received from expression (21) for different values of material parameters σ and b, are shown in Fig. 2. The curve $e_c(h, \sigma, b)$ determines the boundary of Freedericksz transition in FN under the combined action of electric and magnetic fields. The region under the curve corresponds to the unperturbed state of FN with planar texture and orthogonal coupling between the director and magnetization ($\mathbf{n} \perp \mathbf{m}$), the region above the curve $e_c(h, \sigma, b)$ corresponds to the perturbed state.

As is seen from Fig. 2, the electric Freedericksz field e_c increases with the growth of magnetic field h from the value $e_c^{LC} = \pi$, corresponding to the threshold field in a pure nematic [2], to the maximum value e_m and then monotonically decreases to zero. A characteristic feature of the phase diagrams depicted in Fig. 2 is the presence of reentrant phase transitions in FN, conditioned by competition between quadrupole electric [$\sim \varepsilon_0 \varepsilon_a (\mathbf{n} \cdot \mathbf{E})^2$] and magnetic [$\sim \mu_0 \chi_a (\mathbf{n} \cdot \mathbf{H})^2$] mechanisms of influence on NLC-matrix, and dipole [$\sim \mu_0 M_s f\, \mathbf{m} \cdot \mathbf{H}$] magnetic field influence on the ferroparticles. For given values of coupling energy σ and parameter b (characterizing the mechanism of the magnetic field influence on FN) there exists the range of the electric field values $e_c^{LC} \le e \le e_m$, when the growth of magnetic field strength h generates the sequence of orientational transitions from the nonuniform state (provided by the electric Freedericksz transition) to the uniform and back to the nonuniform state. In this case, since the $e_c(h, \sigma, b)$ is a single-value function of the magnetic field h the reentrant orientational transitions can no longer be caused by the electric field e.

The width δ of the area of the reentrant orientational transitions can be found analytically. For this purpose, from relation (21) we find maximum value e_m of electric field strength in the curve $e_c(h, \sigma, b)$. As a result, we obtain

$$\delta = e_m - \pi, \qquad e_m = \sqrt{\pi^2 - \frac{(q - 2\sigma)^4}{9b^2 q^2} + \frac{2\sigma(q - 2\sigma)^2}{6Q\sigma + (q - 2\sigma)^2}}, \tag{22}$$

where $q = \sqrt[3]{\sigma^2(8\sigma + 27b^2 + 3b\sqrt{3(16\sigma + 27b^2)})}$.

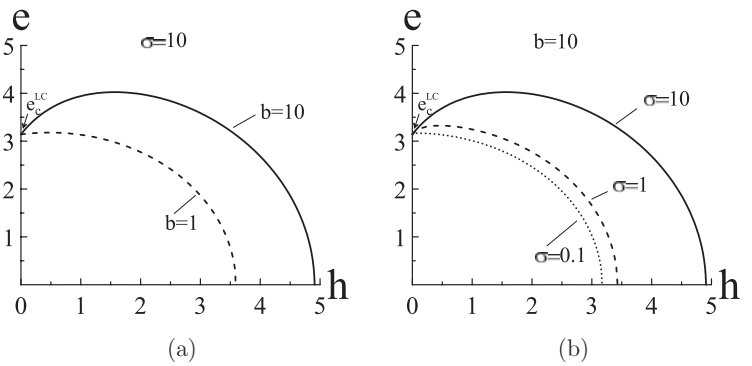

Figure 2. Phase diagrams (e, h) for different values of parameters σ and b; here $e_c^{LC} = \pi$ is the electric Fréedericksz field in pure LCs.

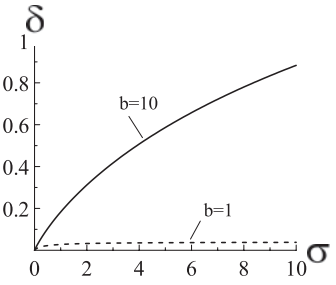

Figure 3. The width $\delta = e_m - e_c^{LC}$ of the area of the reentrant orientational transitions in FN as a function of coupling energy σ.

The function $\delta(\sigma, b)$ determining the width of the area of reentrant orientational transitions is shown in Fig. 3. It is seen that the range of the electric field values, allowing the reentrant uniform phase of FN, increases with the increasing of the coupling energy σ and parameter b and does not depend on the segregation degree, characterized by $\varkappa$ (3).

4. Orientational structure of a ferronematic

The results of numerical simulations of Eqs. (8), (14) – (17) are presented in Figs. 4 – 7. These equations determine the angles $\varphi(z)$ and $\psi(z)$ of the director and the magnetization rotation and the magnetic particles distribution function $f(z)$ in the FN layer.

In weak magnetic field $\mathbf{H}$ without electric field ($\mathbf{E} = 0$) the orientational structure of FN in the layer has planar uniform texture with orthogonal coupling between the director and magnetization ($\mathbf{n} \perp \mathbf{m}$). However this state becomes unstable when the magnetic field exceeds some threshold value h_c. The increase in a magnetic field in ferronematic leads to orientation distortions, that is the magnetic Freedericksz transition occurs (Figs. 4 and 5, $e = 0$). As is known the first-order phase transition is characterized by existence of metastable states (see Fig. 5, curve e = 0, where the angle φ_0 of the director orientation is multiple-valued function of magnetic field). On the contrary, in the second-order transition there is

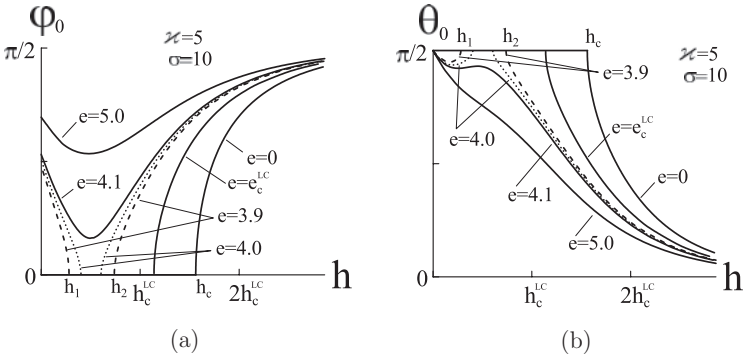

Figure 4. (a) The angle φ_0 of the director orientation, and (b) the angle $\theta_0 = \pi/2 + \psi_0 - \varphi_0$ between the director and magnetization in the middle of the layer as functions of a magnetic field strength h for $\varkappa = 5$, $\sigma = 10$, $k = 1.56$, and $b = 10$; here $h_c^{LC} = \pi$ is the magnetic Freedericksz field in pure nematic; $h_1 = 0.88$, $h_2 = 2.33$.

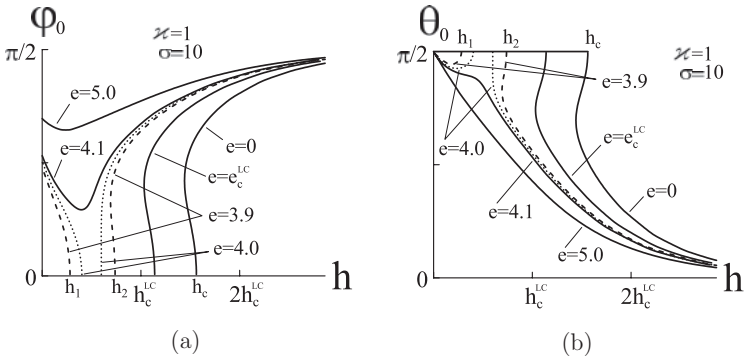

Figure 5. (a) The angle φ_0 of the director orientation, and (b) the angle $\theta_0 = \pi/2 + \psi_0 - \varphi_0$ between the director and magnetization in the middle of the layer as functions of a magnetic field strength h for $\varkappa = 1$, $\sigma = 10$, $k = 1.56$ and $b = 10$; here $h_c^{LC} = \pi$ is the magnetic Freedericksz field in pure nematic; $h_1 = 0.88$, $h_2 = 2.33$.

no metastable states (see Fig. 4, curve e=0, where φ_0 is single-valued function of magnetic field). For weak magnetic segregation ($\varkappa \geq \varkappa^*$) the orientational transition between the uniform and the nonuniform states is the second-order phase transition (Fig. 4, $e = 0$), like the classical Freedericksz transition in pure nematic liquid crystals [2]. For strong magnetic segregation ($\varkappa < \varkappa^*$) the Freedericksz transition in ferronematics becomes the first-order

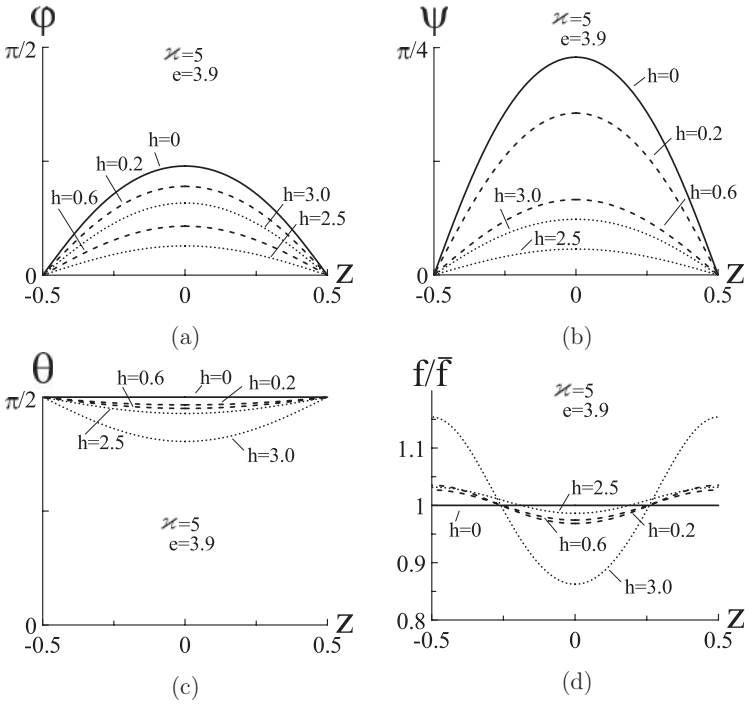

Figure 6. Orientational and concentrational distributions within the ferronematic layer for $\varkappa = 5$, $\sigma = 10$, $k = 1.56$, $b = 10$, $e = 3.9$, and different magnetic field strength h.

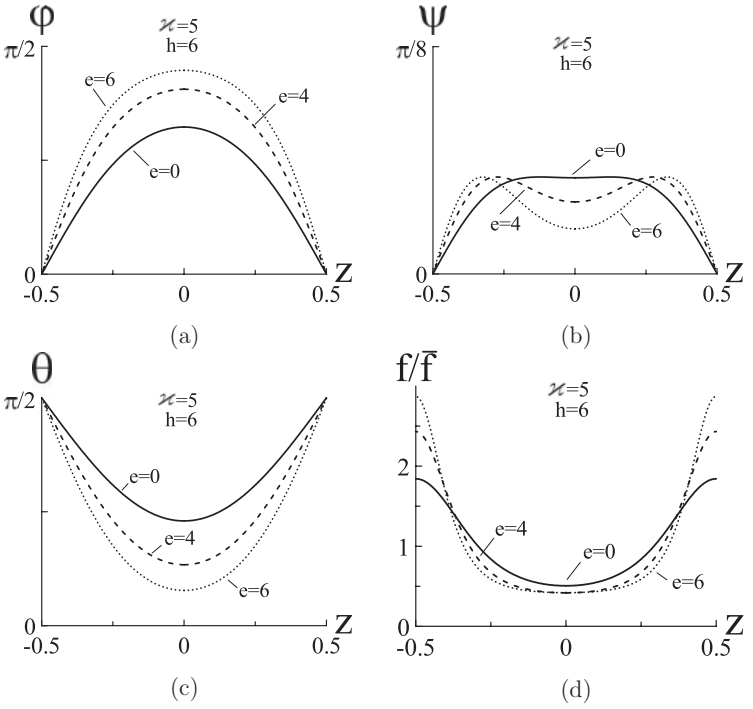

Figure 7. Orientational and concentrational distributions within the ferronematic layer for $\varkappa = 5$, $\sigma = 10$, $k = 1.56$, $b = 10$, $h = 6$, and different electric field strength e.

one (Fig. 5, $e = 0$). Here $\varkappa^*$ is the tricritical segregation parameter, which determines boundary between the first- and second-order magnetic transitions in ferronematics without electric field ($e = 0$):

$$\varkappa^* = \frac{\left[h_c^2 - \pi^2\right]^2}{3bh_c s^2(2-s)^2 + 4\pi^2 k}, \quad h_c = \sqrt{\pi^2 + sbh_c}.$$

We derived these analytical expressions in one of our previous works [14], where tricritical behavior of magnetic Freedericksz transition was analyzed in detail.

At nonzero electric field **E** (codirectional to magnetic field **H**) Eqs.(7) – (10) admit as before a uniform solution $\varphi(z) = \psi(z) = 0$, corresponding to the planar texture of FN. If electric field strength $e < e_c^{LC}$ (i.e., less than the critical electric Freedericksz field in pure LC) the increase in the magnetic field h also induces the Freedericksz transition (see Figs. 4 and 5). In this case, the critical magnetic field h_c decreases (see Fig. 2), since the electric field assists the transition of FN in a nonuniform state.

Texture of FN behaves interestingly in the electric field in the range $e_c^{LC} \le e \le e_m$ corresponding to the area of ambiguity in the phase diagrams (Fig. 2). At $e \ge e_c^{LC}$ uniform texture FN becomes unstable and the electric Freedericksz transition occurs (see Figs. 4 and 5, e = 3.9). A magnetic field leads to a decrease in deviation of the system from the uniform orientational state (the decrease in the director and magnetization angles within the layer) and the increase in the concentration of magnetic particles near the boundaries of the layer (Fig. 6, $h = 0, 0.2, 0.6$). The subsequent increase in its intensity to $h = h_1$ returns the orientation structure of FN to the initial uniform state (Figs. 4 and 5, e = 3.9).

This effect is conditioned by the competition between the quadrupole [$\sim \varepsilon_0 \varepsilon_a (\mathbf{n} \cdot \mathbf{E})^2$] mechanism of an electric field influence on the NLC-matrix and dipole [$\sim \mu_0 M_s f\, \mathbf{m} \cdot \mathbf{H}$] influence of a magnetic field on magnetic moments of ferroparticles. In the presence of a magnetic field $\mathbf{H}$ the magnetization vector $\mathbf{m}$ tends to orient along the field direction. The magnetization vector $\mathbf{m}$ is coupled [$\sim \frac{w}{d} f(\mathbf{n} \cdot \mathbf{m})^2$] with the director $\mathbf{n}$, and therefore it restores the latter to the initial state. However, this state becomes unstable at the magnetic field strength $h = h_2$ and the secondary Freedericksz transition occurs (Figs. 4 and 5, e=3.9). In this case, the magnetic quadrupole [$\sim \mu_0 \chi_a (\mathbf{n} \cdot \mathbf{H})^2$] mechanism, effecting the liquid-crystalline matrix, begins to dominate the magnetic dipole [$\sim \mu_0 M_s f\, \mathbf{m} \cdot \mathbf{H}$] mechanism. Further increase in the magnetic field strength leads to the increase in the angles of rotation of the director and magnetization within the layer and decrease in the concentration of magnetic particles in the center of the layer (see Fig. 6, $h = 2.5, 3.0$). Particles move into sample regions, where they are closely aligned along the magnetic field direction, and at the same time they have the most favorable orientations (close to the orthogonal) relatively to the local direction of $\mathbf{n}$. In the considered geometry these preferable regions locate near the sample boundaries.

In addition to reducing of the critical magnetic field, the electric field can lead to the change of the phase-transition character. In the absence of an electric field ($e = 0$) at strong magnetic segregation ($\varkappa < \varkappa^*$) the orientational transitions in FN are the first-order ones. But at the presence of an electric field these transitions can be either first- (Fig. 5, $e = e_c^{LC}$ and 3.9) or second-order (Fig. 5, $e = 4.0$) ones.

Thus, in the considered range of electric field strength ($e_c^{LC} \le e \le e_m$) the magnetic field generates the sequence of reentrant orientational transitions from the nonuniform state to a uniform and back to the nonuniform state. If the electric field strength e exceeds the value e_m, i.e., it lies in the region over the phase curve $e_c(h, \sigma, b)$ corresponding to the perturbed state of FN (Fig. 2), then the variation of the magnetic field does not lead to any threshold effects.

The concentration of magnetic particles and the angles of rotation of the director and magnetization as functions of the electric field at a given magnetic field are shown in Fig. 7. We consider the magnetic field strength $h > h_c^{LC}$. As is seen, the increase in electric field leads to the increase in the director angle φ and the angle θ between the director and the magnetization, besides the concentration f of magnetic particles in the middle of the layer decreases.

5. Conclusion

Within the continuum theory we have studied the combined influence of electric and magnetic fields on the orientational structure of a ferronematic layer taking into account the segregation effects. We have considered a ferronematic layer which is subjected to a codirectional electric and magnetic fields normal to its plane. The anisotropies of a magnetic susceptibility and dielectric permeability are considered to be positive. We have imposed rigid planar coupling conditions of the director at the layer boundaries and soft orthogonal coupling between the director and the magnetization. Numerical calculations for the concentration of magnetic particles and for the angles of the director and magnetization rotation have been performed. We have received orientational and concentrational distributions in a ferronematic layer for different values of the electric and magnetic field strengths, the coupling energy, the Frank elastic constants, and the segregation parameter.

We have theoretically found the reentrant Freedericksz transitions in ferronematic liquid crystals subjected to the electric and magnetic fields. These transitions can be either first- or second-order ones. We have derived the analytical expression for the width of the area of the reentrant orientational transitions. The phase diagram of the Freedericksz transition under the combined action of electric and magnetic fields have been obtained.

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